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18MAT41

**Fourth Semester B.E. Degree Examination**  
**Complex Analysis, Probability and Statistical Methods**

(Common to all Programmes)

Time: 3 Hrs

Max.Marks: 100

**Note: Answer any FIVE full questions, choosing at least ONE question from each module.**  
**Use of statistical tables allowed.**

**Module-1**

1. (a) Show that  $w = f(z) = z + e^z$  is analytic and hence find  $dw/dz$ . (06 Marks)
- (b) Derive Cauchy-Riemann equation in cartesian form. (07 Marks)
- (c) Find the analytic function  $f(z) = u + iv$ , given  $v = [r - (1/r)]\sin \theta, r \neq 0$ . (07 Marks)

**OR**

2. (a) If  $f(z) = u(x, y) + iv(x, y)$  is an analytic function, show that the family of curves  $u(x, y) = c_1$  and  $v(x, y) = c_2, c_1$  &  $c_2$  being constants, intersect each other orthogonally. (06 Marks)
- (b) If  $f(z)$  is analytic, show that  $[(\partial^2/\partial x^2) + (\partial^2/\partial y^2)]|f(z)|^2 = 4|f'(z)|^2$ . (07 Marks)
- (c) Show that the function  $u = \sin x \cosh y + 2 \cos x \sinh y + x^2 - y^2 + 4xy$  is harmonic. (07 Marks)
- Also, determine the corresponding analytic function. (07 Marks)

**Module-2**

3. (a) State and prove Cauchy's theorem. (06 Marks)
- (b) Find the image in the  $w$ - plane bounded by the lines  $x = 1, y = 1, x + y = 1$  under the transformation  $w = z^2$ . (07 Marks)
- (c) Find the bilinear transformation which maps the points  $z = 0, 1, \infty$  into the points  $w = -5, -1, 3$ , respectively. What are the invariant points under this transformation? (07 Marks)

**OR**

4. (a) Evaluate:  $\int_0^{2+i} (\bar{z})^2 dz$  along the line  $x = 2y$ . (06 Marks)

(b) Evaluate:  $\int_C \frac{e^z}{(z+1)(z-2)} dz$  where  $C$  is the circle  $|z|=3$ . (07 Marks)

(c) Find the bilinear transformation which maps the points  $z = 1, i, -1$  into the points  $w = 0, 1, \infty$ , respectively. What are the invariant points under this transformation? (07 Marks)

### Module-3

5. (a) A random variable  $X$  has the following probability function for various values of  $x$ :

$X(=x_i)$	-2	-1	0	1	2	3
$P(x)$	0.1	$k$	0.2	$2k$	0.3	$k$

Find (i) the value of  $k$  (ii)  $P(x < 1)$  (iii)  $P(x \geq -1)$ . (06 Marks)

(b) The probability of germination of a seed in a packet of seeds is found to be 0.7. If 10 seeds are taken for experimenting on germination in a laboratory, find the probability that (i) 8 seeds germinate (ii) at least 8 seeds germinate (iii) at most 8 seeds germinate. (07 Marks)

(c) If the life time of a certain types electric bulbs of a particular brand was distributed normally with an average life of 2000 hours and S.D.60 hours. If a firm purchases 2500 bulbs, find the number of bulbs that are likely to last for (i) more than 2500 hours (ii) less than 1950 hours (iii) between 1900 and 2100 hours. (07 Marks)

### OR

6. (a) The probability density function of a random variable  $X(=x)$  is  $f(x) = \begin{cases} kx^2, & 0 < x < 3 \\ 0, & \text{otherwise} \end{cases}$ . (06 Marks)

Find (i) the value of  $k$  (ii)  $P(1 < x < 2)$  and (iii)  $P(x \leq 1)$ .

(b) In a certain factory turning out razor blades, there is a small chance of 0.002 for a blade to be defective. The blades are supplied in packets of 10. Use Poisson distribution to calculate the approximate number of packets containing (i) no defective (ii) one defective (iii) two defective blades, in a consignment of 10,000 packets. (07 Marks)

(c) The IQ of students in a certain college is assumed to be normally distributed with mean 100 and variance 25. If two students are selected at random, find the probability that (i) both of them have IQ between 102 and 110 (ii) at least one of them have IQ between 102 and 110 (iii) at most one of them have IQ between 102 and 110. (07 Marks)

**Module-4**

7. (a) The ranking of 10 students in two subjects, Field Theory ( $A$ ) and Network Analysis ( $B$ ) are given below:

Roll No.of the student	1	2	3	4	5	6	7	8	9	10
$A$	3	5	8	4	7	10	2	1	6	9
$B$	6	4	9	8	1	2	3	10	5	7

Calculate the rank correlation coefficient.

**(06 Marks)**

- (b) Fit a best fitting equation in the form  $y = ax^b$  for the following data:

$x$	1	2	3	4	5	6
$y$	2.98	4.26	5.21	6.1	6.8	7.5

Calculate the value  $y$  when  $x = 3.5$ .

**(07 Marks)**

- (c) With usual notation, compute  $\bar{x}$ ,  $\bar{y}$  and  $r$  from the following lines of regression:

$$y = 0.516x + 33.73 \text{ and } x - 32.52 = 0.512y + 32.52.$$

**(07 Marks)**

**OR**

8. (a) In a bivariate distribution, it is found that  $\sigma_x = \sigma_y$  and the acute angle between the lines of regression is  $\tan^{-1}(3)$ . Find the correlation coefficient.

**(06 Marks)**

- (b) Find the coefficient of correlate on between the industrial production and export, using the following table:

**(07 Marks)**

<i>Production</i> (in Lakh tons)	55	56	58	59	60	60	60
<i>Exports</i> (in Lakh tons)	35	38	38	39	44	43	45

- (c) Following are the measurements of air velocity ( $x$ ) and evaporation coefficient ( $y$ ) of burning fuel droplets in an impulse engine:

$x$	20	60	100	140	180	220
$y$	0.18	0.37	0.35	0.78	0.56	0.75

Find a best fitting parabola  $y = ax^2 + bx + c$  to the above data and hence estimate  $y$  when  $x = 99$ .

**(07 Marks)**

**Module-5**

9. (a) Determine (i) marginal distribution (ii) covariance between the discrete random variables  $X$  and  $Y$ , using the joint probability distribution:

**(06 Marks)**

$Y \backslash X$	3	4	5
2	1/6	1/6	1/6
5	1/12	1/12	1/12
7	1/12	1/12	1/12

- (b) A die is thrown 9000 times and a throw of 3 or 4 was observed 3240 times. Show that the die cannot be regarded as an unbiased one.

**(07 Marks)**

- (c) A certain stimulus administered to each of the 12 patients resulted in the following change in the blood pressure 5,3,8,-1,3,0,6,-2,1,5,0,4. Can it be concluded that the stimulus will increase the blood pressure? (*Note* :  $t_{0.05}$  for 11 d.f. is 2.201)

**(07 Marks)****OR**

10. (a) Explain the terms: (i) Null hypothesis (ii) Confidence intervals (iii) Type I and Type II errors.

**(06 Marks)**

- (b) It is claimed that a random sample of 49 tyres has a mean life of 15,200 kms. Is the sample drawn from a population whose mean is 15,150 kms and whose standard deviation is 1200 kms? Test the significance at 0.05 level.

**(07 Marks)**

- (c) Fit a binomial distribution for the data and test the goodness of fit given that  $\chi^2_{0.05} = 7.815$  for 3 d.f.: **(07 Marks)**

<i>No. of Heads</i>	0	1	2	3	4
<i>Frequency</i>	122	60	15	2	1

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