Model Question Paper with effect from 2017-18

USN | | | | | | | | | | | **17MAT11**

First Semester B.E.(CBCS) Examination
Engineering Mathematics-I
(Common to all Branches)

Time: 3 Hrs   Max.Marks: 100

Note: Answer any FIVE full questions, choosing at least ONE question from each module.

Module-I

1. (a) Find the n^{th} derivative of $\frac{x}{(2x+3)(3x-5)}$ **(06 Marks)**

   (b) Find the angle between the curves: $r = a(1-\cos\theta)$ and $r = b(1+\cos\theta)$ **(07 Marks)**

   (c) Find the radius of curvature for the curve: $x^3 + y^3 = 3axy$ at $(3a/2, 3a/2)$. **(07 Marks)**

OR

2. (a) If $x = \sin t, y = \cos mt$, prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2 - n^2)y_n = 0$ **(06 Marks)**

   (b) With usual notation, prove that $\tan\phi = r\frac{d\theta}{dr}$ **(07 Marks)**

   (c) Find the radius of curvature for the cycloid $x = a(\theta + \sin\theta); y = a(1-\cos\theta)$. **(07 Marks)**

Module-II

3. (a) Find the Taylors series of $\sin x$ in powers of $\left(x - \frac{\pi}{2}\right)$ up to fourth degree terms. **(06 Marks)**

   (b) If $u = \tan^{-1}\left(\frac{x^3+y^3}{x+y}\right)$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$ **(07 Marks)**

   (c) If $u = yz/x; v = zx/y; w = xy/z$, show that $J[(u,v,w)/(x,y,z)] = 4$. **(07 Marks)**

OR

4. (a) Evaluate $\lim_{x\to 0}\left(\frac{a^x + b^x + c^x + d^x}{4}\right)^{1/x}$ **(06 Marks)**

   (b) Using Maclaurin's series, prove that $\sqrt{1+\sin 2x} = 1 + x - \frac{x^2}{2} - \frac{x^3}{3} + \frac{x^4}{24} + ...$ **(07 Marks)**

   (c) If $u = f(x-y, y-z, z-x)$, prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$ **(07 Marks)**

Module-III

5. (a) A particle moves along a curve whose parametric equations are $x = e^{-t}, y = 2\cos 3t, z = 2\sin 3t,$ where t is the time. Find the velocity and acceleration in the direction of $\vec{i} - 3\vec{j} + 2k$, at $t = 0$. **(06 Marks)**

   (b) Find $div\vec{F}$ and $curl\vec{F}$ at the point $(1,-1,1)$ where $\vec{F} = grad\left(x^3 + y^3 + z^3 - 3xyz\right)$ **(07 Marks)**

   (c) Show that vector field $\vec{F} = \left[(xi + yj)/(x^2 + y^2)\right]$ is both solenoidal & irrotational **(07 Marks)**

OR

6. (a) For any scalar field and vector field, prove that $Div\left(\phi \vec{F}\right) = \phi\, Div\vec{F} + (grad\phi)\cdot \vec{F}$ **(06 Marks)**

   (b) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at $(2,-1,2)$ **(07 Marks)**

   (c) If $\vec{F} = (x + y + az)i + (bx + 2y - z)j + (x + cy + 2z)k$, find a,b,c such that $\vec{F}$ is irrotational. **(07 Marks)**

Module-IV

7. (a) Obtain reduction formula for $\int_0^{\pi/2} \sin^n x dx, (n > 0)$. **(06 Marks)**

   (b) Solve the differential equation: $r\sin\theta - \cos\theta \frac{dr}{d\theta} = r^2$. **(07 Marks)**

   (c) Find the orthogonal trajectory of the family of curves $\frac{x^2}{a^2} + \frac{y^2}{a^2 + \lambda} = 1$. ($\lambda$ -parameter) **(07 Marks)**

OR

8. (a) Evaluate: $\int_0^2 x^2 \sqrt{2x - x^2}\, dx = \frac{5\pi}{8}$ **(06 Marks)**

   (b) Solve the differential equation : $y(2xy + 1)dx - xdy = 0$. **(07 Marks)**

   (c) A bottle of mineral water at a room temperature of $72^0 F$ is kept in a refrigerator where the temperature is $44^0 F$. After half an hour, water cooled to $61^0 F$. What is the temperature of the mineral water in another half an hour? **(07 Marks)**

Module-V

9. (a) Solve the system of equations $83x + 11y - 4z = 95;\ 7x + 52y + 13z = 104; 3x + 8y + 29z = 71$, using Gauss-Seidel method. **(06 Marks)**

   (b) Using Rayleigh's power method, find largest eigen value and eigen vector of the matrix:
   $\begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix}$ by taking $X^{(0)} = [1,0,0]^T$ as initial eigen vector. **(07 Marks)**

   (c) Reduce $8x^2 + 7y^2 + 3z^2 - 12xy + 4xz - 8yz$ into canonical form using orthogonal transformation. Also indicate the nature, index, rank, and signature of the quadratic form. **(07 Marks)**

OR

10. (a) Find the rank of the matrix $\begin{pmatrix} 1 & 1 & 1 & 6 \\ 1 & -1 & 2 & 5 \\ 3 & 1 & 1 & 8 \\ 2 & -2 & 3 & 7 \end{pmatrix}$ **(06 Marks)**

(b) Reduce the matrix $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$ into the diagonal form **(07 Marks)**

(c) Show that the transformation $y_1 = x_1 + 2x_2 + 5x_3$; $y_2 = 2x_1 + 4x_2 + 11x_3$; $y_3 = -x_2 + 2x_3$ **(07 Marks)**
is regular. Write down the inverse transformation.

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